Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Topology

Subject Code: 4SC06TOP1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 22/09/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Topology. **(01)**

b) Let $X = \{1,2,3\}$, write discreate topology on X. **(01)**

c) Define:Usual topology on R **(01)**

d) Consider coffinite topology and co countable topology. Which topology is stronger than the other? **(01)**

e) Give example of Door space. **(01)**

f) Define: Exterior of set. **(01)**

g) Define: Closure of set. **(01)**

h) Is it true that $(A \cap B)^0 = A^0 \cap B^0$? **(01)**

i) True/false: $\bar{\bar{A}} = \bar{A}$. **(01)**

j) Define: Continuous function. **(01)**

k) Define: Homeomorphism. **(01)**

l) Define: Disconnected space **(01)**

m) Define: T_1 space. **(01)**

n) Define: Compact space. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let X be a set. **(05)**
Let $\tau_c = \{U \subset X \mid \text{either} X - U = X \text{or} X - U \text{is countable}\}$then prove that (X, τ_c) is topological space.

b) Let X be topological space. Then prove that, **(05)**
(i) $\emptyset$, X are closed
(ii) Any arbitrary union of open sets in $X$is open.
(iii) Any finite intersection of open sets in X is open.

c) Let X be a non-empty set. Compare the following topology,
(i) Co countable topology (ii) Co – finite topology (iii) Indiscrete topology **(04)**
(iv) Discrete topology

Q-3 **Attempt all questions** **(14)**

a) Let (X, τ) be a topological space &A, B be two subsets of X then prove that, **(06)**

(i) If $A \subset B$ then $extB \subset ext\ A$.
(ii) $ext\ (A \cup B) = ext\ A \cap ext\ B$.
(iii) $ext\ (A \cap B) = ext\ A \cup ext\ B$.

b) Let (X, τ) be a topological space and Y be a non empty subset of X then prove that the collection $\tau_y = \{U \cap Y \mid U \in \tau\}$ is a topology on Y. **(06)**

c) Let $X = R$ and $A = (2,3)$, find A^0 and A'. **(02)**

Q-4 Attempt all questions **(14)**

a) Let X be a topological space and A be a subset of X. Then prove that $\bar{A} = A \cup A'$. **(05)**

b) Let X and Y be a topological space and $f: X \to Y$, then prove that following are equivalent **(06)**
(i) f is continuous.
(ii) For every subset A of X then $f(\bar{A}) \subset \overline{f(A)}$.
(iii) For every close set B in Y then $f^{-1}(B)$ is closed in X.

c) Let (X, τ) be disconnected space and τ' is finer than τ. Prove that (X, τ') is disconnected. **(03)**

Q-5 Attempt all questions **(14)**

a) Let X, Y, Z be topology space and $f: X \to Y$ and $g: Y \to Z$ are continuous functions then prove that $gof: X \to Z$ is continuous. **(05)**

b) Prove that indiscrete topology is not T_0 space. **(05)**

c) Is R compact space with usual topology? Justify your answer. **(04)**

Q-6 Attempt all questions **(14)**

a) Prove that every closed subset of compact space is compact. **(07)**

b) Prove that continuous image of compact space is compact. **(07)**

Q-7 Attempt all questions **(14)**

a) Prove that every compact subset of T_2 space is closed. **(07)**

b) Prove that continuous image of connected space is connected. **(07)**

Q-8 Attempt all questions **(14)**

a) State and prove Heine Borel theorem. **(10)**

b) Prove that every subspace of T_1 space is T_1 space. **(04)**